# Constrained Diffusion Limited Aggregation in 3 Dimensions

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## Abstract

Diffusion Limited Aggregation (DLA) has usually been studied in 2 dimensions as a model of fractal growth processes such as river networks, plant branching, frost on glass, electro-deposition, lightning, mineral deposits, and coral. Here the basic principles are extended into 3 dimensions and used to create, among other things, believable models of root systems. An additional innovation is a means of constraining the growth of the 3 dimensional DLA by a surface or containing it within a vessel.

## DLA in 3 dimensions

The rules for forming DLA structures are very simple and were first introduced in 2 dimensions by T.A. Witten and L.M. Sander around 1981 [1]. A particle is introduced into an environment at a random position, it moves around randomly (for example: Brownian motion) until it encounters the existing structure (initially just a single stationary particle) at which stage it permanently adheres at the point of contact and becomes part of the DLA structure. As an example of DLA in the plane see figure 1, it can be readily appreciated that the growth tends to occur at the tips of the structure and the chances of a particle reaching the inner parts of the structure is lower. This characteristic operates across scales and gives rise to the fractal nature of DLA structures. The fractal dimension of an ideal DLA simulation has been calculated as 1.71 but this seems to be sensitive to the implementation details [2]. A reasonable approximation to physical systems is met when the density of particles is low enough that they can be modeled one particle at a time, in other words, the probability of interaction between unattached particles is negligible.

There are many refinements [3] discussed in the literature that improve the performance and allow for variations in the geometric and statistical structure [4]. For example, once a particle strays too far from the current fixed structure, it may be discarded since the probability of it finding it's way back is low. In most implementations the particle is not discarded but rather the random walk is constrained to lie within the maximum radius of the current structure. A variable often introduced into DLA growth simulation is a "stickiness" value, when a particle encounters the fixed structure it only adheres with some probability. If the stickiness probability is low then denser structures result since the particle is able to penetrate deeper into the current structure. A detailed discussion of these and other options is outside the scope of this discussion except to note that the efficiencies and variations that apply in 2 dimensions can generally be extended into 3 dimensions.

Extending diffusion limited aggregation to 3 dimensions [5] is relatively straightforward but there are some important differences and new possibilities not available in 2 dimensions. In 2 dimensions it is common practice to grow structures on a discrete grid, namely the square pixel grid that will eventually form the DLA image. A significant departure of the algorithm implemented here is that it does not extend the pixel concept to form the DLA on a cubic grid of finite resolution in 3 dimensions but rather on a continuum. A particle adheres to the existing structure if it comes within some minimum distance of any part of the existing structure. The process of adhering to the existing DLA structure involves extending a branch from the closest position on the existing structure to the new particle position. The minimum distance before the particle adheres becomes a convenient variable that controls the structural detail. As in 2 dimensions, a probability of adhesion (stickiness) can be used to control the density of the structure.

The general shape of the DLA structure can be controlled by varying the spatial distribution at which the particles randomly enter the system. The classical 2 dimensional DLA introduces the particles at random positions on a circle with a radius just larger than the bounds of the existing structure, this leads to structures that tend to grow evenly. The equivalent in 3 dimensions is employed in figure 2 where new particles are introduced to the simulation randomly on the surface of a sphere. Directed growth as illustrated in figure 3 is achieved by introducing particles along one axis. Other ways of controlling the overall form of the DLA is to apply additional forces on the structure such as gravity or wind, or to slowly rotate the DLA structure while it is being formed.

## Constraint surfaces

In order to support very general constraint surfaces, a simple but industry standard file format was chosen to describe the geometry, namely the STL format. The STL format was developed for stereo lithography, it is in common usage by the rapid prototyping industry and as such it is supported as an export format by many 3D modeling packages. The STL format describes a surface as a collection of triangles in 3 dimensions along with a normal that determines which side of the triangle is “inside” and which is “outside”, this is an important requirement for rapid prototyping of closed solids and is also used in the software outlined here.

Constraining the DLA growth by a surface or solid described by an STL dataset occurs at the stage when the new particle is to be attached to the existing structure. If the branch that is to be extended from the existing structure to the new particle intersects a triangle then it either isn’t added, or the intersection with the triangle is calculated and a branch is added that just touches the constraint surface. In the former case the simulation continues, that is, the particle continues on the random walk. This approach tends to result in DLA structures that appear to follow and fill the constraint surface, a desirable visual effect when simulating root structures where one expects them to follow the shape of a vessel.

## Results

Figure 4 shows a very simple constraint surface made up of a cylinder with an open top. The particles are added randomly on a sphere about the center of the cylinder. In the implementation discussed here no consideration is given to the cylinder until the stage at which the particle is close enough to the existing structure to adhere. This is an important implementation detail from a performance perspective. Since most of the time is spent in the random walk phase and the constraint surface may be made up of a very large number of triangles, testing for intersection at every stage of the random walk would be prohibitively time consuming.

Figure 5 shows a more complicated constraint surface where the DLA structure is constrained to lie within a 3D scanned bust of a human subject with added antlers. In this case the surface consists of approximately 25000 triangular faces. This example also illustrates another implementation feature, namely the growth of the DLA structure from multiple (in this case 2) independent seeds. An additional possibility when using a constraint surface is to relate the spatial distribution of new particles to the geometry of the surface in some way. In this example the distribution of the new particles matches the radial density of the head model. This results in more particles being added around the antlers which otherwise would probably not have filled given the narrow gap between them and the head.

The examples shown here are all rendered using PovRay, a ray tracing package with a powerful scene description and scripting language that is both free and supported across multiple computer platforms. PovRay supports a high level geometric primitive called a “sphere-sweep” defined as the solid space a sphere of variable radius occupies as it moves along a spline curve. This primitive is used to define each branch strand, the positions at which the particles adhere during the DLA aggregation define the control points of the spline curve and the radius of the sphere is varied to give a branch thickness that decreases with distance from the start of the branch. The exact branch thickness variation and appearance of the branches can thus be determined and changed without recreating the structure.

## Conclusion

The techniques described here provide the basis for an efficient DLA growth simulation which can be constrained by a surface described with a simple and well documented 3 dimensional file format that is supported by many 3D modeling packages. The DLA structures formed using the algorithm described here look like natural branching structures and could thus be used to form models for a number of computer graphics applications. In addition the structures are grown in time and therefore can be used to create animation sequences. Changing the parameters of the simulation process and rendering the resulting geometry using different materials can result in structures that have a strong visual similarity with a range of branching structures, such as coral (thick short branches) or arteries/veins (thin elongated branches).

The author would like to acknowledge Philippe Vranjes for the model of the human bust and the initial motivation to develop the software described. The software to create these structures is available by request as well as by download from the authors web site [http://astronomy.swin.edu.au/~pbourke/fractals/dla3d].

References

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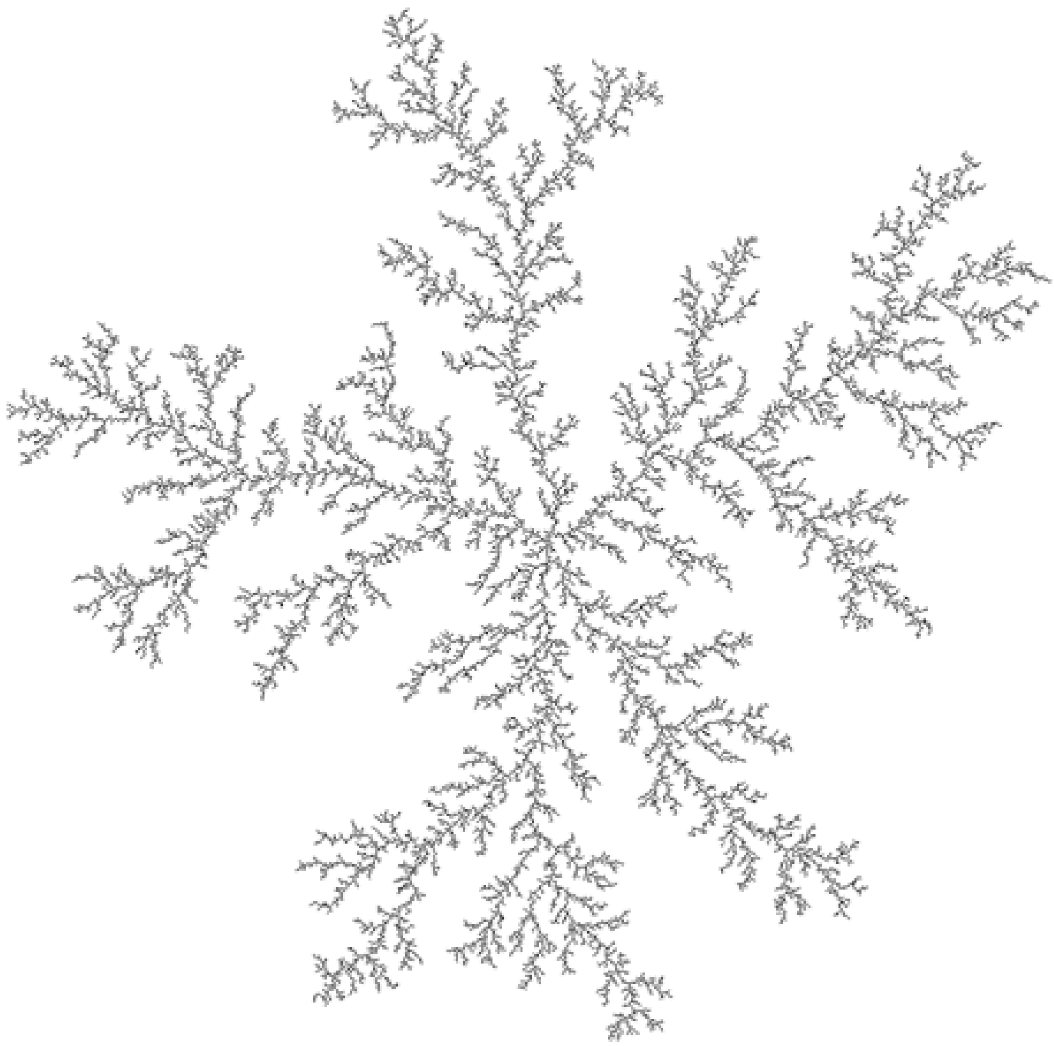

Figure 1. Traditional 2 dimensional DLA structure with a stickiness of 0, containing 30,000 particles.

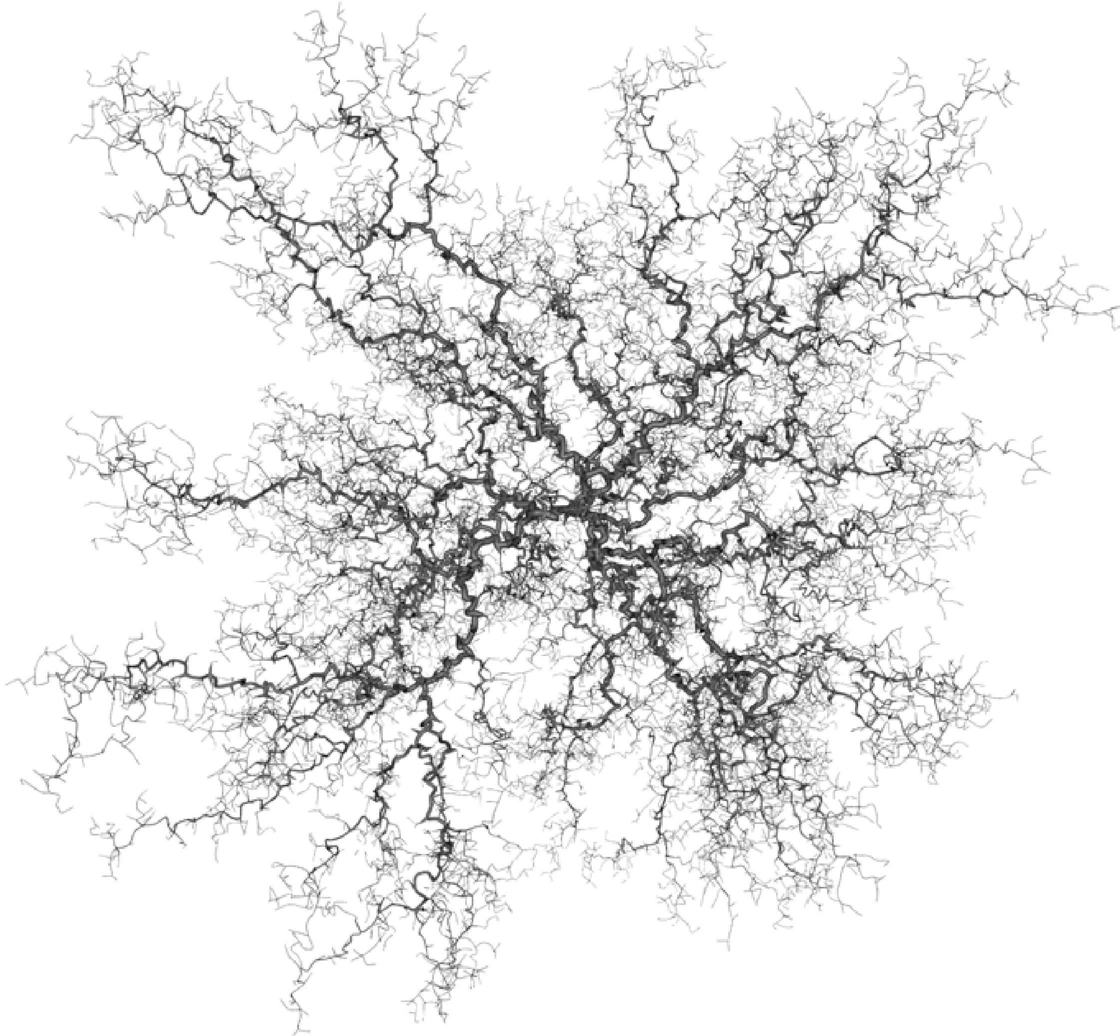

Figure 2: Diffusion Limited Aggregation extended into 3 dimensions, without constraints and particles introduced randomly on the surface of a sphere surrounding the initial seed particle.

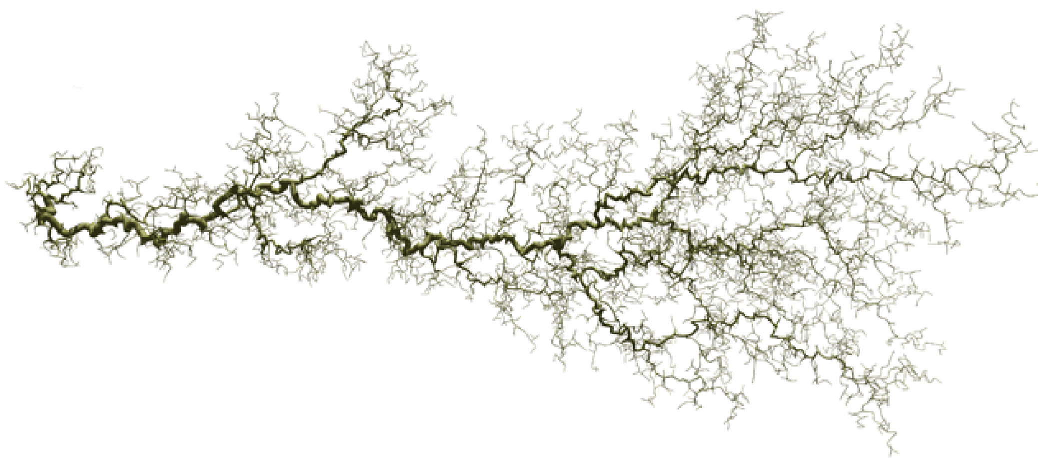

Figure 3: Directed DLA growth formed by introducing particles randomly within a small disk positioned along one of the coordinate axes. The initial seed particle is located to the left of the structure.

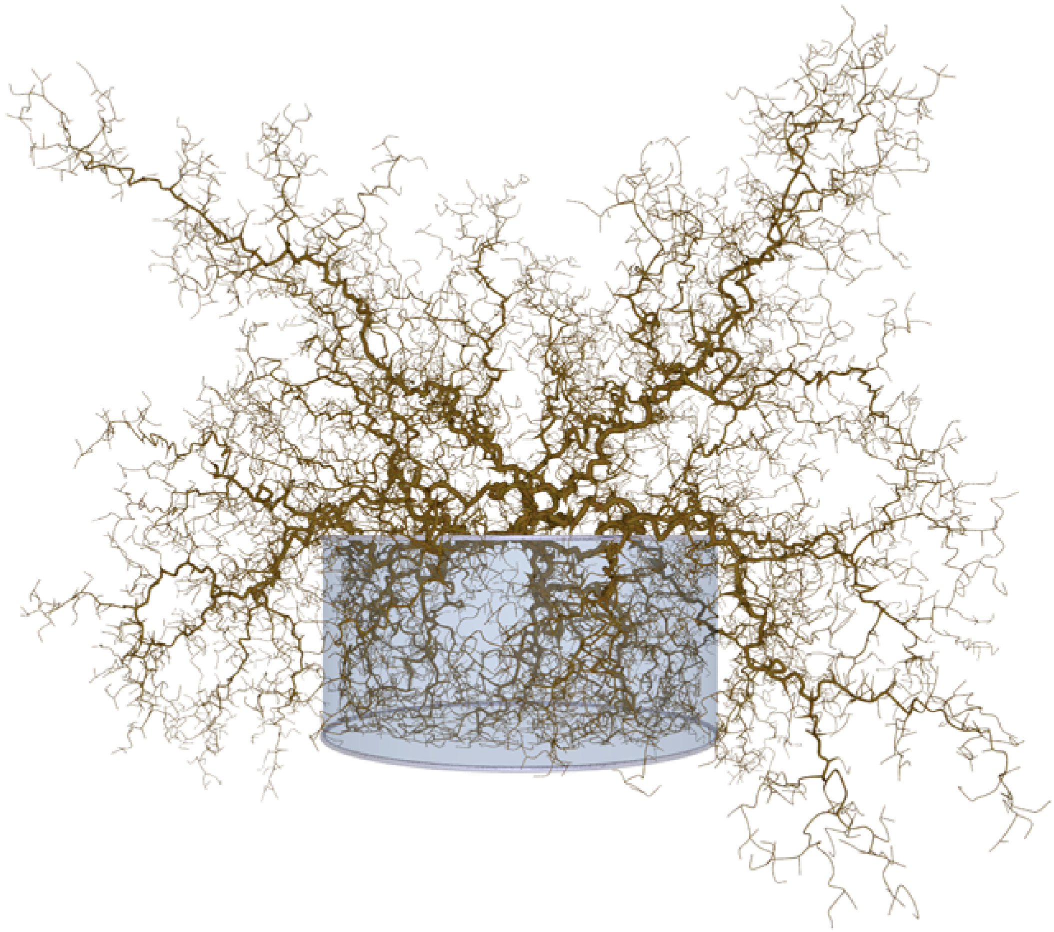

Figure 4: DLA constrained within an open can (a cylinder sealed at the base but open at the other end). Particles are added randomly on the surface of a sphere with the origin at the center of the cylinder. The initial seed particle is located at the center of the top of the cylinder.

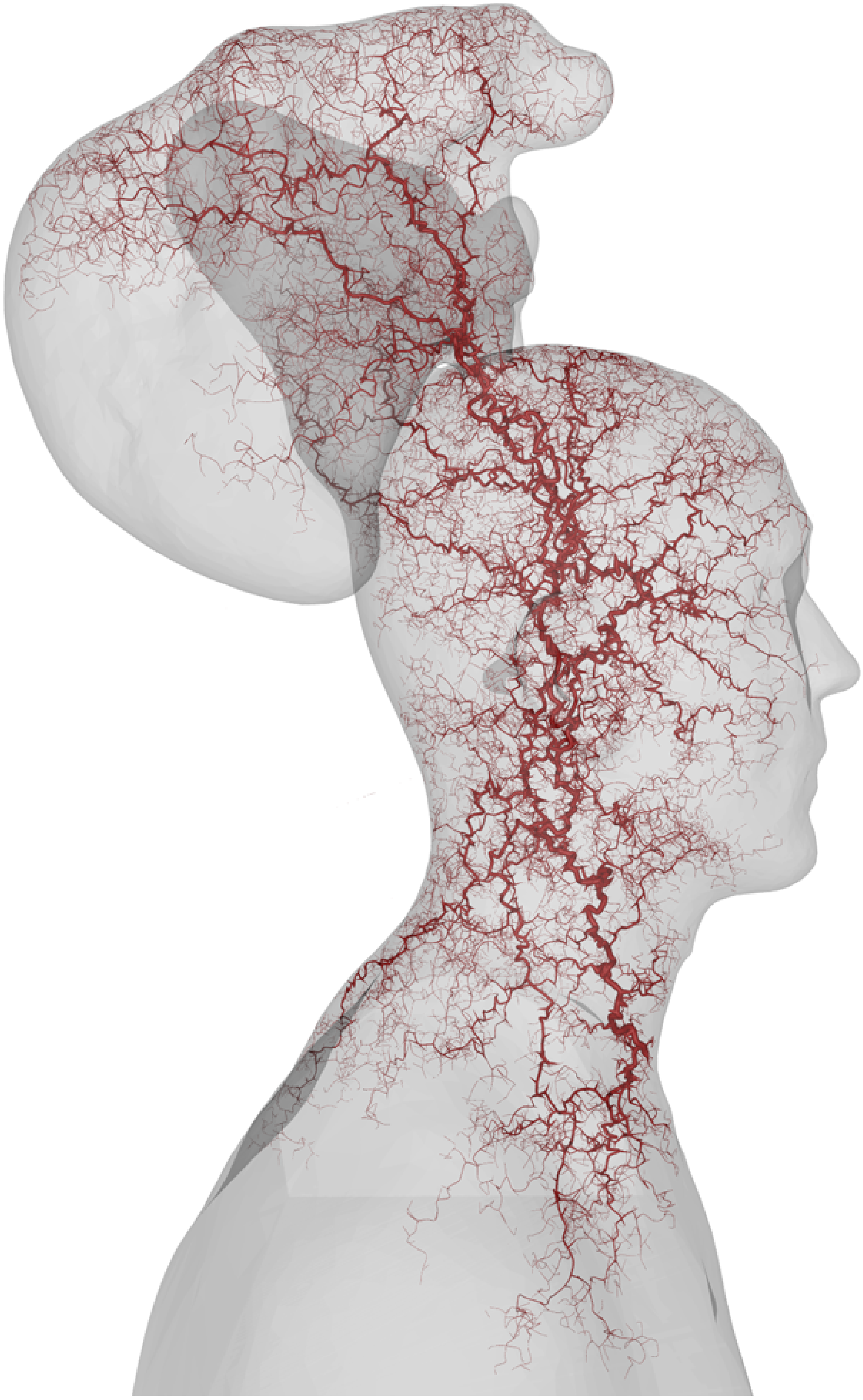

Figure 5: DLA constrained to lie within a 3 dimensional, digitally scanned human bust (with antlers). Particles are added randomly with a distribution proportional to the radial mesh density of the constraint surface. There are two initial seed particles, one in the throat and the other just behind the eyes.