Ellipse's Perimeter Estimations

Abstract

It is well known, since 1959, that the perimeter of an ellipse is expressed by

L=4*a*(1+(b/a)^s)^(1/s) (1)
Where s=r*t/(r-t) and t=t(b/a)

The expression (1) is the equivalent of the finite elliptic integrals on the positive Cartesian. It concern all the astroids, ellipse included, expressed by

(x/a)^r+(y/b)^r=1 (2)

where (a,b) are the semi-axes length and (r=power of the astroid)
Case (r=2) is the case of an ellipse

In these papers we will crack (1) for (r=2) in order to get various accuracies.
The accuracy is defined as the value of the error % in the whole range of (b/a)

(L estimated-L exact)/L estimated=error % (3)

say (b/a)=TANgent. (it's value).And consider (b>a)

We write, for the unit total arc length of any astroid, on the positive Cartesian

L1=(1+TAN^s)^(1/s) (4)

We do not use eccentricity; it has no meaning, in general.

Cracking

(s=Constant version)
we know L1exact=(Pi/2)*R

from L1=(1+(R/R)^s)^(1/s)=(Pi/2)*R we get s.min=ln(2)/ln(Pi/2)

s.min=1,53492853566138.. (5)
This gives the error graph (Fig.I)
Max.error %=0,003605937….at TAN=5,006784983..

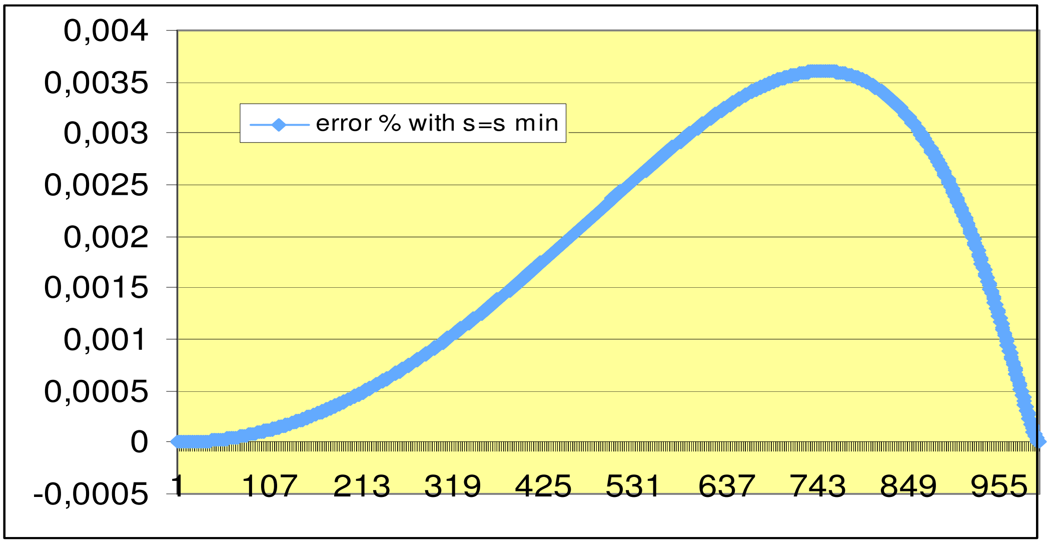

Fig.I Error % graph for (1<TAN<infinity)

(s=Linear variable version)

we know L1Exact=636,62517105986… for TAN(89,91o)=636,6192488…
from L1 max=(1+TAN.max^s)^(1/s) we get

s.max=1,71122902010321….then, according the value of TAN

s=s.min+(s.max-s.min)/(TAN.max-1)*(TAN-1)

s=1,53492853566138+0,000277355*(TAN-1) (6)

This gives the error graph (Fig.II)
Max.error %=0,003477761…….at TAN=4,966157118..

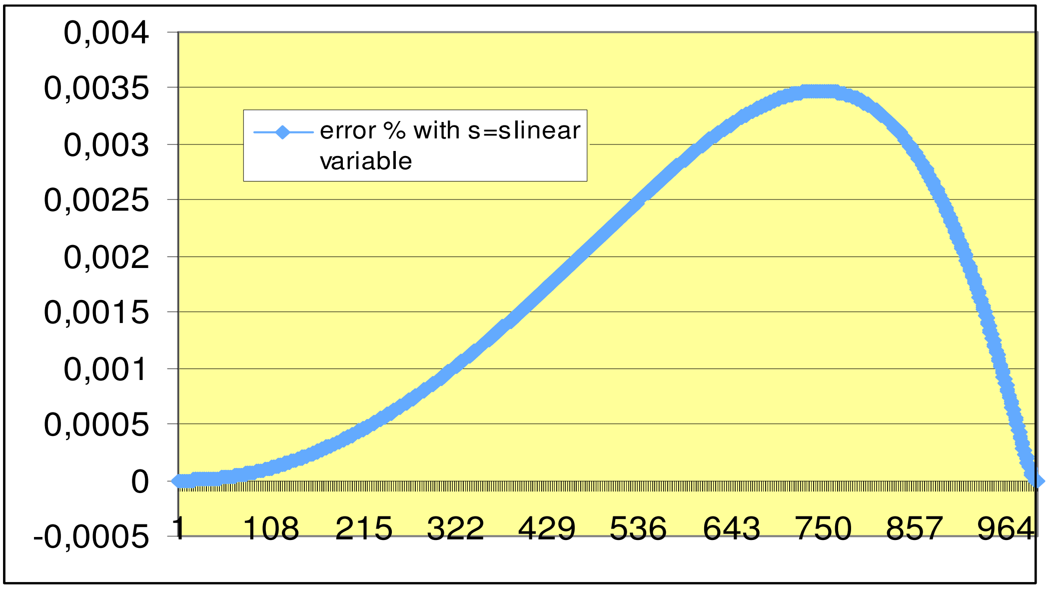

Fig.II Error % graph for (1<TAN<infinity)

(s=Power version,with p=2,98 constant)
we know all L1Exact
an (s) data is obtained.Fig.III

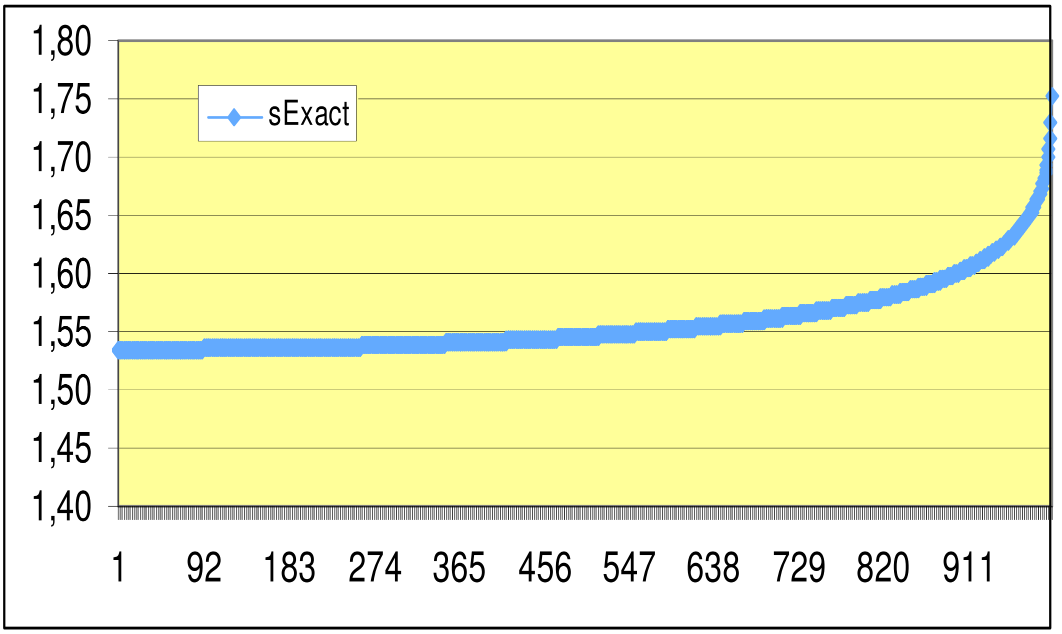

Fig.III s data graph
By similarity,we write an (s Mod) overlapping the (s Real)
sMod=astroid+polynome
sMod=d1+b1*(1-(x-c1)/a1)^p)^(1/p)+(F+m1*x^v1+n1*x^w1)
where ,for 1000 lines of evaluation,

Angle step	=(90o-45o)/1000
x	=(Angle-45)/Angle step
Angle o	=(45+angle step*x)
ATAN	=angle o*Pi/180
Angle o	=ATAN*180/Pi

Fig.IV shows the overlapping with the following parameter values. Table I

parameters	**values**
a1	1000
(smin-sMax)=b1	-0,193967895182134
c1	0
d1	0,000000000000000
p	***2,980000000000000***
sMax=F	1,728896430843500
m1	0,000000000000000
v1	1
n1	0
w1	1,00
smin=	1,534928535661380

Table I

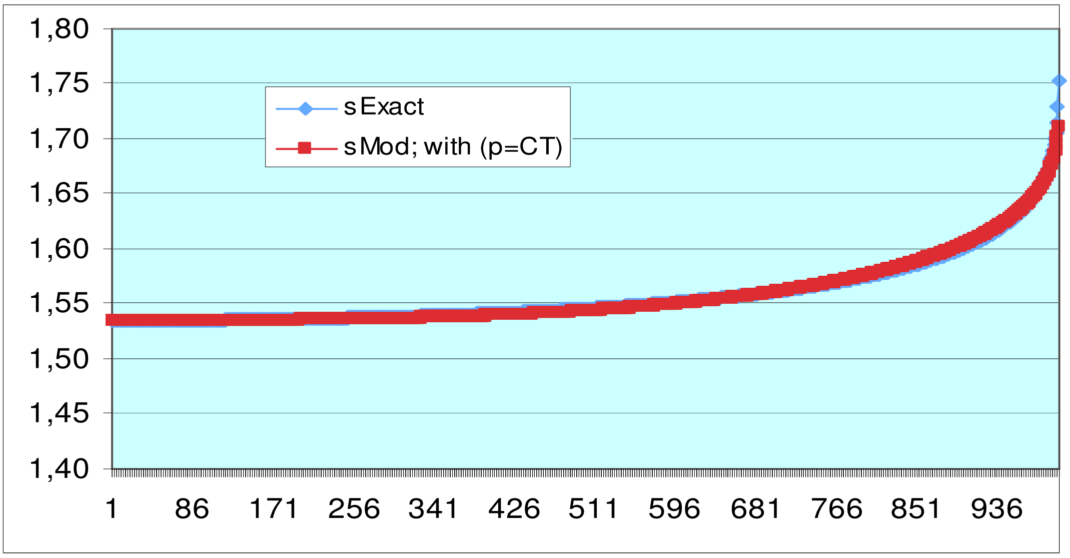

Fig.IV Overlapping of sExact&sMod with p=Constant

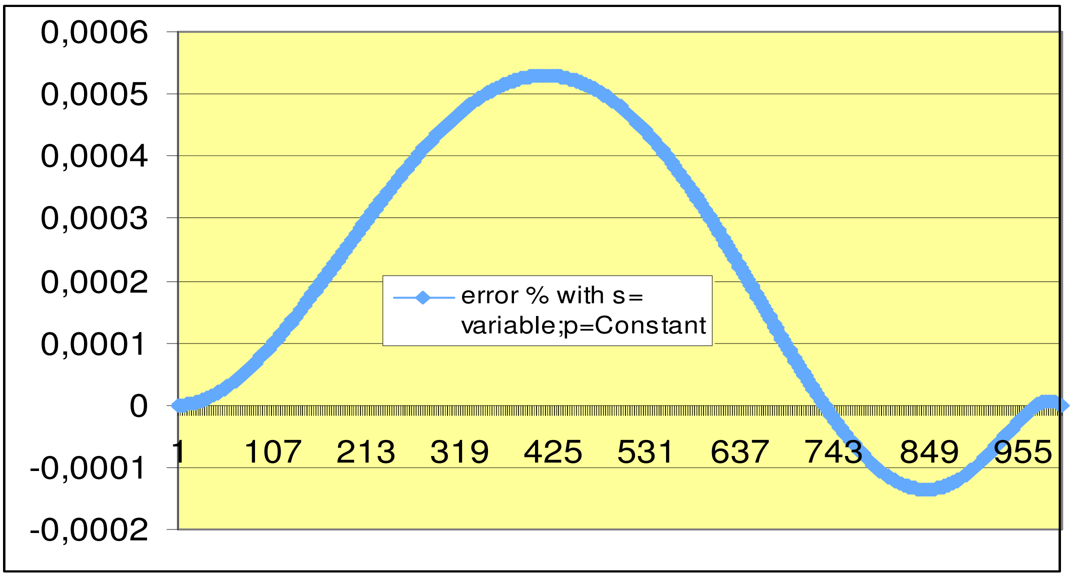

Fig.V Error % for (1<TAN<infinity) (power p=2,98)
Max error %=0,000529940456316923… at TAN=2,021125559…

(s power version, with p=variable)
we know all L1 exact
we know the overlapping sExact&sMod
we write error % for the overlapping=0,this is:
(sMod-sExact)/sMod=0
which gives a new value for (p) at every different TAN.
The graph of this new (p) looks like an astroid.
We write a pMod for this pExact data,with the following parameters.Table II

pMod=d2+b2*(1-((x-c2)/a2)^q)^(1/q)+(G+m2*x^v2+n2*x^w2)

parameters	values
a2	500
b2	0,3
c2	500
d2	0,000
q	6
G	**1,965**
m2	**0,000996000**
v2	1
n2	0
w2	1

Table II

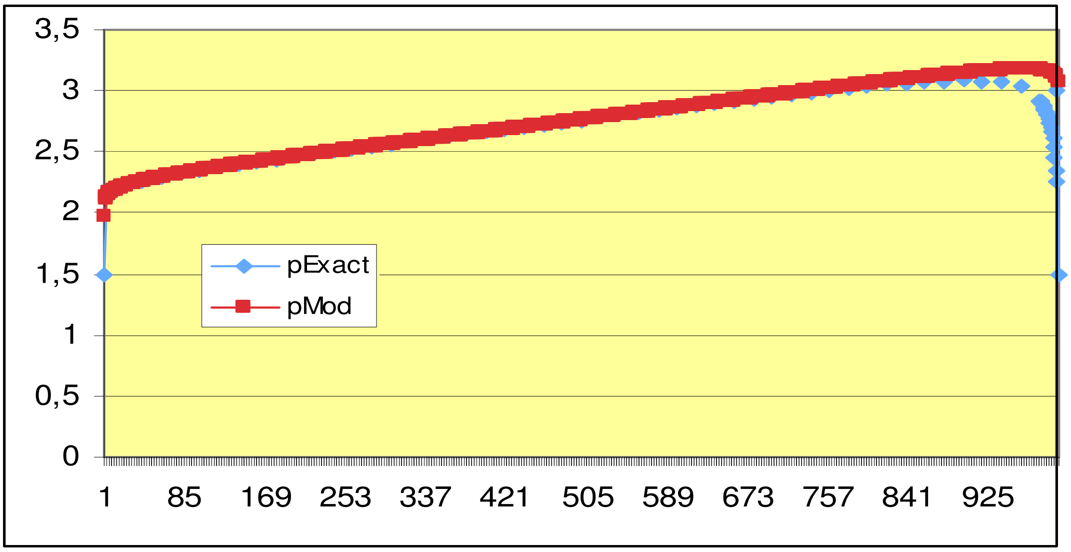

Fig.VI overlapping of pExact&pMod.

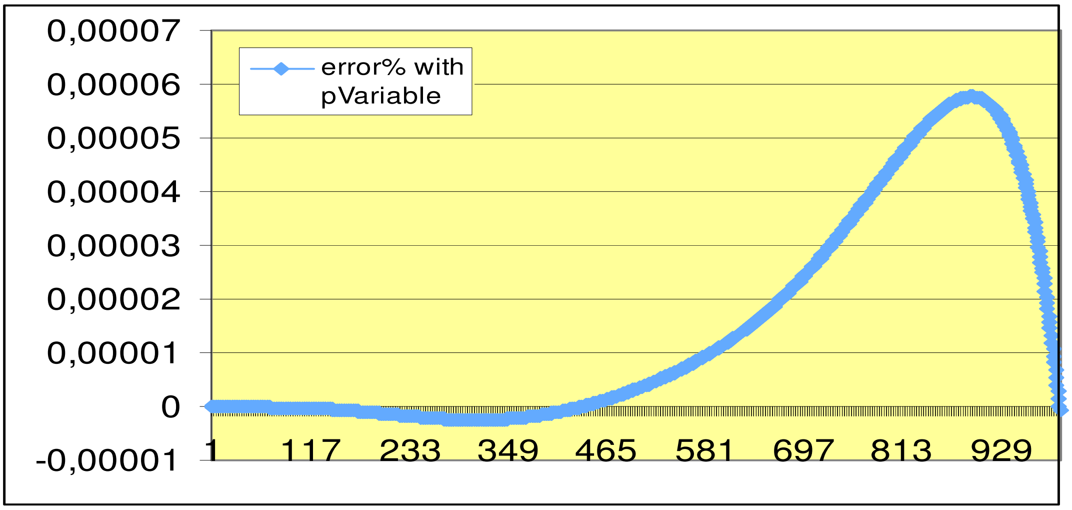

Fig.VII Error % for (1<TAN<infinity) (p variable)
Max.Error %=0,000057694942746…. at TAN=12,0985895…

(s power version, with p=variable, corrected)

we know all L1 exact

we know the overlapping pExact&pMod

we write error % for this overlapping=0, that is:

(pMod-pExact)/pMod=0 and

we attack the parameter (b2,m2,G) of pMod) consecutively

All these parameters have an astroidal looking.

parameters	values
a3	**500**
b3	**0,340000**
c3	**500**
d3	**0,000000**
r	**23**
H	**0,638500**
m3	**0,000040**
v3	**1,000000**
n3	**-0,000001**
w3	**1,600000**

Table III. b2 parameter correcting pMod

parameters	values
a4	500
b4	-0,000906000100
c4	500
d4	0
t	12
J	0,000092
m4	0,0000000000
v4	1
n4	0
w4	1

Table IV.m2 parameter correcting pMod

parameters	values
a5	1000
b5	-0,797500000
c5	0
d5	0
u	8,37
K	1,171710000000000
m5	0,000001000000000
v5	0
n5	-0,0000011
w5	1,4

TableV. G parameter correcting pMod

Now, with these corrections we have

pMod=d2+b2Mod*(1-((x-c2)/a2)^q)^(1/q)+(GMod+m2Mod*x^v2+n2*x^w2)
sMod=d1+b1*(1-((x-c1)/a1)^pMod)^(1/pMod)+(F+m1*x^v1+n1*x^w1)

Error % is reduced as shown on Fig.VII

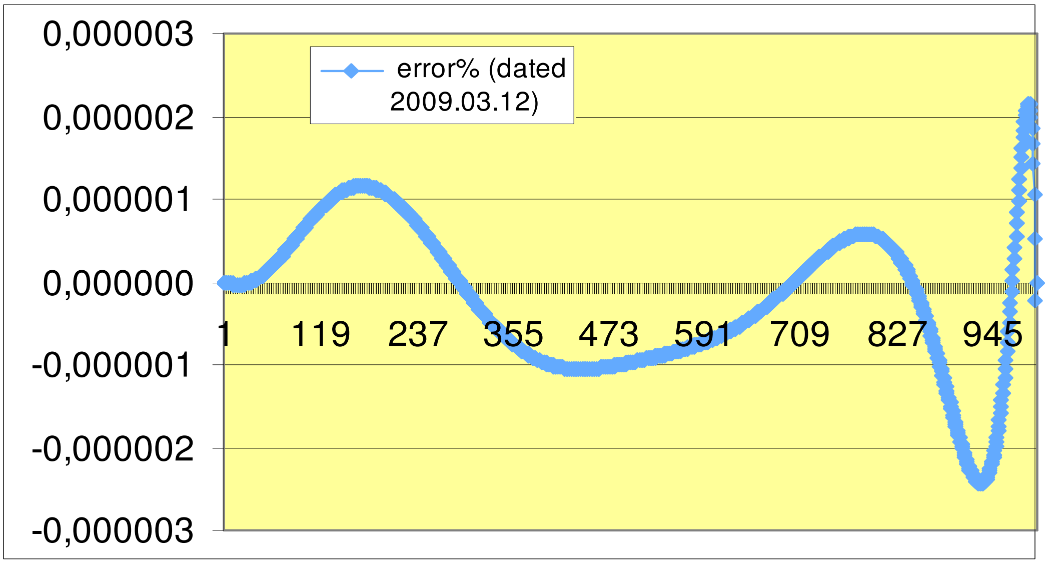

Fig:VII Error % for (1<TAN<infinity) (p corrected, coarse)
Max.error %=-0,000002432846011… at TAN=18,43467933..

It is possible to correct (pMod) finely. Then the error will go down.

A numerical example

Evaluate the perimeter of the ellipse (a=1;b=5).We calculate

TAN=b/a	=5
ATAN	=1,373400767..
Angle o	=78,69006753 o
x	=748,66817..

we use (x) value in b2,m2,G,in their formula ….. we get

b2Mod	=0,288740099	
m2Mod	=0,000997983	
GMod	=1,948787387	then,
pMod	=2,99518385	
sMod	=1,567203335	are evaluated

Use (sMod) in L1=(1+TAN^sMod)^(1/sMod)

Find L1Estimated	=5,25251332979251
L1 Exact	=5,25251113492227 (with tools,mathematica,or ….)
Error %	=0,000000418041087..

Conclusion

The cracking method is valid for all astroid.
For all (0<r<infinity)
For all (1<TAN<infinity)
The equivalency formula of the elliptic finite integrals K=(1+TAN^s)^(1/s) means total arc length, or area of the astroids, or …what else.
It explains also an enlarged Thales theorem!
It has nothing common with Hölder mean.
One may go further for better accuracy.

Necat Tasdelen
necattasdelen@ttmail.com